

HOMEWORK 1

BMEN 3010

1] VISCOELASTIC EFFECTS ARISE FROM WATER REDISTRIBUTING THROUGH THE TISSUE AS IT DEFORMS. APPLYING A

DEFORMATION SLOWLY ALLOWS WATER TO FLOW AND EQUILIBRATE AS FORCES ARE APPLIED, REDUCING RATE-DEPENDENT EFFECTS.

2] A) COLLAGEN IS RANDOMLY ORGANIZED IN SKIN, SUCH THAT (IN-PLANE), PROPERTIES DO NOT DEPEND ON ORIENTATION OF LOADING.

LIGAMENTS HAVE HIGHLY ALIGNED COLLAGEN, SUCH THAT THEY ARE STIFFER ALONG THE DIRECTION OF ALIGNMENT THAN PERPENDICULAR TO IT.

B) EVOLUTIONARILY, IT WOULD BE COSTLY TO REINFORCE A TISSUE WITH EXTRACELLULAR MATRIX IN A DIRECTION WHERE IT DOES NOT SEE MUCH LOADING. TISSUE FORM IS GENERALLY WELL OPTIMIZED TO FUNCTION. SKIN

IS BIAXIALLY LOADED, AND HAS MORE
ISOTROPIC COLLAGEN ORGANIZATION TO
ACCOMMODATE THIS.

3

$$A) \delta_{22} = \delta_{11} + \delta_{22} + \delta_{33}$$

$$= 1 + 1 + 1$$

$$= 3$$

B) $\epsilon_{ijk} \epsilon_{kji}$ HAS 6 NONZERO
COMBINATIONS. EACH WILL
INVOLVE A PRODUCT OF
CYCLIC AND ANTICYCLIC
PERMUTATIONS:

$$= \epsilon_{123} \epsilon_{321} + \epsilon_{312} \epsilon_{213} + \epsilon_{231} \epsilon_{132}$$

$$+ \epsilon_{321} \epsilon_{123} + \epsilon_{132} \epsilon_{231} + \epsilon_{213} \epsilon_{312}$$

$$= 1(-1) + 1(-1) + 1(-1) + 1(-1) + 1(-1) + 1(-1)$$

$$= -6$$

C) $\delta_{ik} a_k$ by contraction $\Rightarrow a_i$

$$\text{also: } \delta_{i1} a_1 + \delta_{i2} a_2 + \delta_{i3} a_3 = a_i$$

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$$\begin{aligned}
 \underline{u} \times \underline{v} &= u_i v_j \underline{e}_i \times \underline{e}_j \\
 &= u_i v_j \epsilon_{ijk} \underline{e}_k \\
 &= v_j u_i \epsilon_{ijk} \underline{e}_k \\
 &= -v_j u_i \epsilon_{jik} \underline{e}_k \quad \left. \begin{array}{l} \epsilon_{ijk} = -\epsilon_{jik} \end{array} \right\} \\
 &= -\underline{v} \times \underline{u}
 \end{aligned}$$

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$$\underline{u} = 3\underline{e}_1 + 5\underline{e}_2 - 7\underline{e}_3$$

$$\underline{v} = 6\underline{e}_1 + 3\underline{e}_3$$

$$\underline{u} + \alpha \underline{v} \perp \underline{v} \Rightarrow (\underline{u} + \alpha \underline{v}) \cdot \underline{v} = 0$$

$$0 = \underline{u} \cdot \underline{v} + \alpha \underline{v} \cdot \underline{v}$$

$$= 3(6) + (-7)(3) + \alpha [6^2 + 3^2]$$

$$0 = -3 + \alpha (45)$$

$$\boxed{\alpha = \frac{1}{15}}$$

[6]

$$\underline{u} = 3\underline{e}_1 + 5\underline{e}_2 - 7\underline{e}_3$$

$$|\underline{u}| = (\underline{u} \cdot \underline{u})^{1/2}$$

$$= (9 + 25 + 49)^{1/2}$$

$$(83)^{1/2}$$

$$\hat{\underline{u}} = \frac{\underline{u}}{|\underline{u}|} = 83^{-1/2} [3\underline{e}_1 + 5\underline{e}_2 - 7\underline{e}_3]$$

[7]

A) $a_i = F_{ij} b_j + c_i$ where
 $i = \text{free}$
 $j = \text{dummy}$

B) $\underline{a} = \underline{F} \cdot \underline{b} + \underline{c}$